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Three-Dimensional Oscillatory Piecewise Continuous-Kernel Function Method—Part II: Geometrically Continuous Wings

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The rapid convergence and high-accuracy characteristics of the three-dimensional piecewise continuous-kernel function method (PCKFM) are tested in the present work. The large number of numerical examples which are included herein is aimed at showing the computational efficiency of the method. Three-dimensional wings with no geometrical discontinuities along their spans (discontinuities are permitted at the wing root only) are treated. Problems associated with geometrical discontinuities (such as break points, control surfaces, etc.) are presented in Part III of this work.

Introduction

THE basic ideas of the piecewise continuous-kernel function method (PCKFM) have been outlined in Refs. 1 and 2 and shown to yield excellent results for the two-dimensional case. The initial problems associated with the extension of the method to three-dimensional flows was treated in Ref. 3. These problems included the choice of the spanwise pressure polynomials, the determination of the spanwise collocation points, and the treatment of some numerical integration techniques made necessary by the method. In this paper, application is made of the ideas and methods presented in Refs. 1-3 with the object of testing the resulting accuracy and the computational efficiency of a three-dimensional PCKFM. A large number of numerical examples is presented in this work. All of the examples relate to wings with no geometrical discontinuities along their spans (discontinuities are permitted at wing root only). This class of wings is referred to as single-box wings. Wings with geometrical discontinuities are treated in a subsequent paper.[4]

Comparison between Results Obtained by Applying Different Methods on Single-Box Wings

In this section, the results obtained for single-box wings will be compared with those obtained using other methods. The objective of the comparison is to establish the validity of both the conceptual and numerical methods introduced for the simple case of single-box wings before embarking on complex examples with multigeometrical singularities. All cases of steady and unsteady flows with and without compressibility effects will be considered.

In an attempt to obtain an indication on the relative computational effort involved, a doublet-lattice program was written, using the same kernel function routine as used in the new PCKFM program. Since both programs were run on the same computer (IBM 370/168), a comparison between the computer's CPUs gives a clear indication regarding the relative computational time involved by the two methods.

Examples Involving Steady Flows

Table 1 shows a comparison of the aerodynamic coefficients computed by various methods for a circular wing (with $A\!\!R = 1.273$ in a steady incompressible flow). It can be seen that the convergence of the present method is extremely rapid and that two orthogonal pressure polynomials in each direction (i.e., chordwise and spanwise) seem to yield excellent results. Table 2 shows a similar comparison relating to a rectangular wing with $A\!\!R = 2$ in steady incompressible flow. Here again, the present method yields excellent results with two pressure polynomials in each direction. The effectiveness and the rapid convergence of the present method are once again demonstrated. Table 3 shows similar results pertaining to a delta wing with $A\!\!R = 2$. The convergence of the aerodynamic coefficients for a rectangular wing with extreme aspect ratios is demonstrated in Table 4. It can be seen that the results relating to the wing with the $A\!\!R = 10$ are very close to those obtained using Glauert's lifting line theory, and the results relating to the wing with $A\!\!R = 0.1$ are very close to those obtained using the slender wing theory. Hence the validity of the present method spans over a very wide range of aspect ratios.

Examples Involving Unsteady Flows

Table 5 shows a comparison of aerodynamic coefficients computed by the PCKFM, Hsu's method[5] and experimental results for a 45 deg swept untapered wing in both unsteady incompressible and compressible flows. Tables 6 and 7 show a comparison of the aerodynamic coefficients computed by the present method and the doublet-lattice method for a CORNELL wing (see Fig. 1). In this case, each wing is considered as a separate box with separate leading-edge, trailing-edge and wing-tip singularities. The number of pressure polynomials in the fore and aft wings is shown in the tables. As can be seen, the convergence of the results obtained by the present method is very rapid and the CPU time required is considerably less than that required by the doublet-lattice method. Finally, Table 8 shows a comparison of the aerodynamic coefficients computed by various methods for a CORNELL wing in unsteady compressible flow. These results supplement those shown earlier and confirm the rapid convergence and high-accuracy characteristics of the PCKFM.

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Table 1 Comparison of aerodynamic coefficients computed by various methods for a circular wing in steady incompressible flow

Method of computation	No. of pressure polynomials Chordwise	Spanwise	C_{L_α}	$C^a_{M_\alpha}$	CPU, s
PCKFM	2	2	−1.818	0.948	2.50
PCKFM	3	3	−1.800	0.931	5.12
Kinner (Ref. 7)	–	–	−1.817	0.936	–
Schad & Krienes (Ref. 8)	–	–	−1.798	0.932	–
Van Speigel (Ref. 9)	–	–	−1.790	0.933	–
Watkins et al. (Ref. 6)	–	–	−1.791	0.939	–

[a]The moment is about the mid ℄ chord with reference chord = 1 and ℄ chord = 2.

Table 2 Comparison of aerodynamic coefficients computed by various methods for a rectangular wing in steady incompressible flow

Method of computation	No. of pressure polynomials Chordwise	Spanwise	C_{L_α}	$C^a_{M_\alpha}$	CPU, s
PCKFM	2	2	−2.478	−1.048	2.38
PCKFM	3	3	−2.480	−1.037	4.82
Lan (Ref. 10)	8	15	−2.471	−1.035	–
VLM (Ref. 11)	6	20	−2.524	−1.067	–
NLR (Ref. 12)	4	15	−2.474	−1.036	–
NPL (Ref. 12)	4	15	−2.475	−1.036	–
BAC (Ref. 12)	4	13	−2.474	−1.036	–
Wagner (Ref. 13)	–	–	−2.478	−1.036	–

[a]The moment is about the wing's leading edge with reference chord = 1 and ℄ chord = 2.

Table 3 Comparison of aerodynamic coefficients computed by various methods for a delta wing (Æ = 2) in steady incompressible flow

Method of computation	No. of pressure polynomials Chordwise	Spanwise	C_{L_α}	$C^a_{M_\alpha}$	CPU, s
PCKFM	2	2	−2.291	−2.683	2.37
PCKFM	3	3	−2.167	−2.507	4.87
Lan (Ref. 10)	3	35	−2.197	−2.568	–
Lamar (Ref. 14)	–	–	−2.239	−2.576	–
VLM, $M = 0.1$ (Ref. 11)	6	20	−2.204	−2.593	–
Wagner (Ref. 13)	–	–	−2.216	−2.595	–
Experiment (Ref. 14)	–	–	−2.119	−2.531	–

[a]The moment is about wing's apex with reference chord = 1 and ℄ chord = 2.

Table 4 Effect on aerodynamic coefficients of varying the aspect ratio of a rectangular wing in steady incompressible flow

Method of computation	No. of pressure polynomials Chordwise	Spanwise	C_{L_α}	$C^a_{M_\alpha}$	CPU, s
	Rectangular wing Æ = 10				
PCKFM	3	3	−4.925	−2.388	4.39
PCKFM	3	4	−4.922	−2.386	5.80
PCKFM	3	5	−4.923	−2.381	7.43
Lifting line	–	–	–	–	–
Glauret (Ref. 15)	–	–	−5.040	−2.520	–
	Rectangular wing Æ = 0.1				
PCKFM	3	3	−0.158	−0.015	–
PCKFM	4	3	−0.157	−0.010	–
PCKFM	5	3	−0.157	−0.009	–
Wing with low	–	–	–	–	–
Æ (Ref. 16)	–	–	−0.157	−0.000	–

[a]The moment is about wing's leading edge with reference chord = 1 and ℄ chord = 2.

Table 5 Comparison of aerodynamic coefficients computed by PCKFM and Hsu's method and experimental results for a swept untapered wing in unsteady flow ($k = 0.4$)

Method of computation	No. of pressure polynomials Chordwise	Spanwise	C_{L_α} (deg)	$C^a_{M_\alpha}$ (deg)	C_{LH} (deg)	C^a_{MH} (deg)	CPU, s
			Incompressible flow (M = 0)				
PCKFM	2	2	2.421 (−147)	0.749 (−109)	0.879 (−76)	0.181 (−51)	4.35
PCKFM	3	3	2.457 (−146)	0.757 (−108)	0.877 (−75)	0.182 (−50)	9.92
Hsu (Ref. 5)	–	–	2.644 (−143)	0.785 (−103)	0.927 (−75)	0.171 (−48)	–
Experiment (Ref. 17)	–	–	2.300 (−152)	0.680 (−126)	1.000 (−76)	0.320 (−72)	–
			Compressible flow ($M = 0.8$)				
PCKFM	2	2	2.892 (−155)	0.996 (−111)	1.025 (−82)	0.241 (−49)	6.15
PCKFM	3	3	2.938 (−154)	1.031 (−110)	1.025 (−81)	0.250 (−47)	14.52
Hsu (Ref. 5)	–	–	3.078 (−152)	0.961 (−107)	1.069 (−81)	0.226 (−45)	–

[a]The moment is about mid ℄ with reference chord = 2 and ℄ chord = 2. The wing oscillates about 70% of ℄ chord (measured from wing apex). The reduced frequency k is based on the reference semichord.

Table 6 Comparison of aerodynamic coefficients computed by PCKFM and doublet-lattice method for a CORNELL wing in unsteady[c] ($k = 2.623$) incompressible flow ($M = 0$)

PCKFM No. of pressure polynomials[a] Chordwise	Spanwise	C_{L_α} (deg)	$C^b_{M_\alpha}$ (deg)	CPU, s	Doublet-lattice method No. of boxes Chordwise	Spanwise	C_{L_α} (deg)	$C^b_{M_\alpha}$ (deg)	CPU,s
2	2	6.0624 (−89.06)	4.1699 (−75.37)	13.93	5	12	6.2182 (−86.92)	4.2832 (−74.09)	200.83
3	3	6.2278 (−88.41)	4.2565 (−74.70)	35.84	6	12	6.2687 (−87.19)	4.3215 (−74.21)	293.12
3	4	6.2508 (−88.33)	4.2758 (−74.63)	47.6	7	12	6.3004 (−87.36)	4.3452 (−74.30)	404.41
3	5	6.2521 (−88.28)	4.2774 (−74.61)	62.02	8	12	6.3208 (−87.46)	4.3602 (−74.35)	534.94
4	3	6.2157 (−88.48)	4.2476 (−74.78)	54.29	8	8	6.4285 (−87.37)	4.4442 (−74.43)	248.91
5	3	6.2122 (−88.52)	4.2435 (−74.77)	77.87	–	–	– –	– –	–
		C_{LH}	C_{MH}				C_{LH}	C_{MH}	
2	2	7.8472 (−49.79)	4.3768 (−36.28)	–	5	12	7.8758 (−48.28)	4.4409 (−35.61)	–
3	3	7.9127 (−49.13)	4.4208 (−35.58)	–	6	12	7.9371 (−48.46)	4.4799 (−35.66)	–
3	4	7.9313 (−49.02)	4.4402 (−35.46)	–	7	12	7.9754 (−48.56)	4.5041 (−35.70)	–
3	5	7.9299 (−49.00)	4.4419 (−35.44)	–	8	12	8.0001 (−48.64)	4.5195 (−35.72)	–
4	3	7.9012 (−49.20)	4.4135 (−35.66)	–	8	8	8.1219 (−48.70)	4.6028 (−35.98)	–
5	3	7.9000 (−49.21)	4.4095 (−35.67)	–	–	–	– –	– –	–

[a]Number of pressure polynomials is identical for the two lifting surfaces (for both the present method and the doublet-lattice method). [b]The wing oscillates about the mid ℄ chord of the forward lifting surface, as is the computed moment (reference chord = 1). [c]The reduced frequency $k = \omega/v$ with ℄ chord = 0.45748.

Table 7 Comparison of aerodynamic coefficients computed by the PCKFM and the doublet-lattice method for a CORNELL wing in unsteady[c] ($k = 2.623$) incompressible flow ($M = 0.7$)

PCKFM No. of pressure polynomials[a] Chordwise	Spanwise	C_{L_α} (deg)	$C^b_{M_\alpha}$ (deg)	CPU, s	Doublet-lattice method No. of boxes Chordwise	Spanwise	C_{L_α} (deg)	$C^b_{M_\alpha}$ (deg)	CPU,s
2	2	6.4386 (−103.25)	4.6870 (−88.32)	18.99	5	12	6.5717 (−100.63)	4.7884 (−86.55)	201.05
3	3	6.6310 (−101.99)	4.8078 (−87.98)	48.58	6	12	6.6208 (−101.07)	4.8333 (−86.85)	293.21
3	4	6.6694 (−102.88)	4.8437 (−87.90)	64.95	7	12	6.6524 (−101.34)	4.8617 (−87.03)	406.07
3	5	6.6602 (−102.98)	4.8404 (−88.05)	93.41	8	12	6.6742 (−101.51)	4.8808 (−87.16)	534.37
4	3	6.6112 (−103.06)	4.7923 (−88.07)	74.49	8	8	6.7664 (−101.40)	4.9575 (−87.23)	248.56
5	3	6.6029 (−103.05)	4.7841 (−88.05)	106.04	–	–	– –	– –	–
		C_{LH}	C_{MH}				C_{LH}	C_{MH}	
2	2	8.6539 (−61.81)	5.1124 (−48.48)	–	5	12	8.6333 (−59.96)	5.1482 (−47.38)	–
3	3	8.7536 (−61.43)	5.1900 (−47.96)	–	6	12	8.7007 (−60.26)	5.1984 (−47.57)	–
3	4	8.7881 (−61.29)	5.2245 (−47.81)	–	7	12	8.7437 (−60.34)	5.2299 (−47.68)	–
3	5	8.7768 (−61.42)	5.2233 (−47.98)	–	8	12	8.7733 (−60.56)	5.2513 (−47.76)	–
4	3	8.7312 (−61.51)	5.1747 (−48.05)	–	8	8	8.8806 (−60.65)	5.3294 (−48.08)	–
5	3	8.7236 (−61.50)	5.1677 (−48.04)	–	–	–	– –	– –	–

[a]Number of pressure polynomials is identical for the two lifting surfaces (for both the present method and the doublet-lattice method). [b]The wing oscillates about the mid ℄ chord of the forward lifting surface, as is the computed moment (reference chord = 1). [c]The reduced frequency $k = \omega/v$ with ℄ chord = 0.45748.

Table 8 Comparison of aerodynamic coefficients computed by various methods for a CORNELL wing in unsteady[c] compressible flow

Method of computation	No. of pressure polynomials[a] Chordwise	Spanwise	C_{L_α} (deg)	$C^b_{M_\alpha}$ (deg)	C_{LH} (deg)	C^b_{MH} (deg)	CPU, s
				$k=1, M=0.7$			
PCKFM	2	2	3.0042 (−125.02)	1.8094 (−90.70)	2.1606 (−73.87)	0.9673 (−24.66)	20.10
PCKFM	3	3	3.0298 (−124.04)	1.8157 (−89.68)	2.1576 (−73.26)	0.9666 (−23.82)	50.75
Stark (Ref. 18)	–	–	3.1405 (−115.10)	1.8640 (−87.17)	2.2175 (−50.68)	0.9860 (−20.65)	–
Doublet-lattice	8	12	3.1124 (−117.28)	1.8743 (−89.96)	2.2167 (−53.08)	1.0012 (−24.70)	534.35
				$k=2, M=0.7$			
PCKFM	2	3	5.1444 (−101.59)	3.6201 (−78.06)	5.8515 (−48.46)	3.3097 (−28.80)	27.64
PCKFM	3	3	5.2745 (−100.75)	3.6941 (−78.90)	5.8969 (−47.85)	3.3462 (−28.02)	74.27
Stark (Ref. 18)	–	–	5.5137 (− 96.74)	3.8322 (−77.78)	6.0893 (−43.78)	3.4470 (−24.26)	–
Doublet-lattice	8	12	5.3476 (− 99.46)	3.7720 (−79.39)	5.9665 (−47.17)	3.4130 (−28.17)	534.73
				$k=2.623, M=0.7$			
PCKFM	2	3	6.4386 (−103.25)	4.6870 (−88.32)	8.6539 (−61.81)	5.1124 (−48.48)	31.65
PCKFM	2	3	6.6310 (−102.99)	4.8078 (−87.98)	8.7536 (−61.43)	5.1900 (−47.96)	80.97
Stark (Ref. 18)	–	–	6.9551 (− 99.40)	5.0174 (−85.08)	9.0305 (−57.56)	5.3593 (−44.60)	–
Doublet-lattice	8	12	6.6742 (−101.51)	4.8808 (−87.16)	8.7733 (−60.56)	5.2513 (−47.26)	534.37

[a]Number of pressure polynomials is identical for the two lifting surfaces (for both the present method and the doublet-lattice method). [b]The wing oscillates about the mid ℄ chord of the forward lifting surface, as is the computed moment (reference chord = 1). [c]The reduced frequency $k=\omega/v$ and ℄ chord = 0.45748.

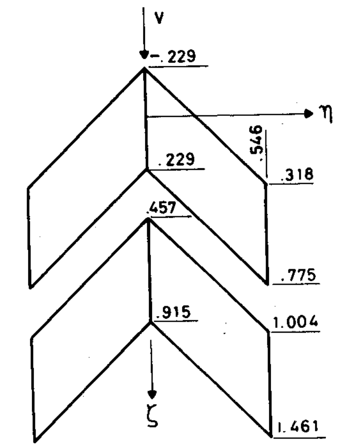

Fig. 1 Plan view of CORNELL wing.

Conclusion

The present method is tested in this paper for single-box wings (i.e., geometrical singularities may be located only at the wing root). Many results are computed which confirm the high accuracy of the method, its rapid convergence, and its considerable reduction in computational time.

Additional results relating to wings with multigeometrical singularities along their span (as for example, in wings with break points and control surface deflections) are presented in a subsequent paper.

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References

[1]Nissim, E. and Lottati, I., "Oscillatory Subsonic Piecewise Continuous Kernel Function Method," National Technical Information Service, Springfield, Va., Paper N77-73109, 1977.

[2]Nissim, E. and Lottati, I., "Oscillatory Subsonic Piecewise Continuous Kernel Function Method," *Journal of Aircraft*, Vol. 14, June 1977, pp. 515-516.

[3]Lottati, I. and Nissim, E., "Three Dimensional Oscillatory Piecewise Continuous-Kernel Function Method—Part I: Basic Problems," *Journal of Aircraft*, Vol. 18, May 1981, p. 346.

[4]Lottati, I. and Nissim, E., "Piecewise Continuous Kernel Function Method—Part III: Wings with Geometrical Discontinuities Along Their Spans," *Journal of Aircraft*, Vol. 17, May 1981, p. 356.

[5]Hsu, P.T., "Some Recent Developments in the Flutter Analysis of Low-Aspect-Ratio Wings," *Proceedings of National Specialists Meeting on Dynamics and Aeroelasticity*, Institute of Aeronautical Sciences, Nov. 1958, pp. 7-26.

[6]Watkins, C.E., Woolston, D.S., and Cunningham, H.J., "A Systematic Kernel Function Procedure for Determining Aerodynamic Forces on Oscillating or Steady Finite Wings at Subsonic Speeds," NASA TR-48, 1959.

[7]Kinner, W., "Die Kreisförmige Tragfläche auf Potential Theorestischer Grundlage," *Ing.-Archiv*, Vol. VIII, No. 1, Feb. 1937, pp. 47-80.

[8]Schade, Th. and Krienes, K., "The Oscillating Circular Airfoil on the Basis of Potential Theory," NASA TM 1908, 1947.

[9]Van Spiegel, E., "Theory of the Circular Wing in Steady Incompressible Flow," National Luchtvaartlaboratorium, Amsterdam, NLL-TN F. 189, Jan. 1957.

[10]Lan, C.E., "A Quasi-Vortex-Lattice Method in Thin Wing Theory," *Journal of Aircraft*, Vol. 11, Sept. 1974, pp. 518-527.

[11]Margason, R.J. and Lamar, J.E., "Vortex-Lattice Fortran Program for Estimating Subsonic Aerodynamic Characteristics of Complex Planforms," NASA TN D-6142, Feb. 1971.

[12]Garner, H.C., Hewitt, B.L., and Labrujere, T.E., "Comparison of Three Methods for the Evaluation of Subsonic Lifting-Surface Theory," NLR TN G-65, April 1969.

[13]Wagner, S., "On the Singularity Method of Subsonic Lifting-Surface Theory," *Journal of Aircraft*, Vol. 6, Nov.-Dec. 1969, pp. 549-558.

[14]Lamar, J.E., "A Modified Multhopp Approach for Predicting Lifting Pressures and Camber Shape for Composite Planforms in Subsonic Flow," NASA TN D-4427, July 1968.

[15]Glauert, H., *The Elements of Airfoil and Airscrew Theory*, Cambridge University Press, Cambridge, England, 1959.

[16]Ashley, H. and Landahl, M.T., *Aerodynamics of Wings and Bodies*, Addison-Wesley Publishing Co., Reading, Mass., 1965.

[17]Merbt, H. and Landahl, M.T., "Aerodynamic Forces on Oscillating Low Aspect Ratio Wings in Compressible Flow," Royal Institute of Technology, Aeronautical Div., Stockholm, KTH Aero TN 30, 1953.

[18]Stark, V.J.E., "Application of the Polar Coordinate Method to Oscillating Wing Configurations," SAAB TN 69, 1974.